7.6

#1

x	f(x)
0	1.41421
.25	1.33372
.5	1.26749
.75	1.21341
1	1.16956

$$\text{Trap}(4) = \frac{1}{2}\left(1.41421 + 1.33372 + 1.33372 + 1.26749 + 1.26749 + 1.21341 + 1.21341 + 1.16956\right)\frac{1}{4}$$

$$= 1.27663$$

x	f(x)
.125	1.37204
.375	1.29896
.625	1.23906
.875	1.19032

$$\text{Mid}(4) = \frac{1}{4}\left(1.37204 + 1.29896 + 1.23906 + 1.19032\right)$$

$$= 1.27509$$

$$\text{Simp}(4) = \frac{2\,\text{Mid}(4) + \text{Trap}(4)}{3} = 1.27560$$

If $n=40$,

Trap(40) would be 100 times as accurate.

Simp(40) would be 10,000 times as accurate.

#2 Column 2 is Simp(N) since the values are so similar. Column 4 is Mid(N) + column 1 is Trap(N) since

$$\text{Column } 2 = \frac{2 \cdot \text{column } 4 + \text{column } 1}{3}.$$

That leaves Left(N) as the third column.

#3 $\displaystyle\int_0^6 e^{2x}\,dx = \frac{e^{2x}}{2}\Big|_0^6 = \frac{e^{12}}{2} - \frac{1}{2} = \frac{e^{12}-1}{2}$

$\text{Left}(2) = 3(e^0 + e^6) \approx 1213$

$\text{Right}(2) = 3(e^6 + e^{12}) \approx 489{,}475$

$\text{Trap}(2) = \dfrac{1213 + 489475}{2} \approx 245{,}344$

$\text{Mid}(2) = 3(e^3 + e^9) \approx 24{,}370$

$\text{Simp}(2) = \dfrac{2(24{,}370) + 245{,}344}{3} \approx 98{,}028$

$\text{Error}_L = \dfrac{e^{12}-1}{2} - 1213 \approx 80{,}164$

$\text{Error}_R = \dfrac{e^{12}-1}{2} - 489{,}475 \approx +408{,}098$

$\text{Error}_T = \dfrac{e^{12}-1}{2} - 245{,}344 \approx -163{,}967$

$\text{Error}_M = \dfrac{e^{12}-1}{2} - 24{,}370 \approx 57{,}007$

$\text{Error}_S = \dfrac{e^{12}-1}{2} - 98028 \approx -16{,}651$

#4

Left :
4 places 2 seconds
8 places 2×10^4 seconds $\longleftrightarrow$ $\approx$ 6 hrs
12 places 2×10^8 seconds $\longleftrightarrow$ $\approx$ 6 years
20 places 2×10^{16} seconds $\longleftrightarrow$ $\approx$ 600 million years

Trap: 4 places $\quad$ 2 seconds $\quad$ $n^2 \sim t$, $n \sim t^{1/2}$

8 places $\quad 2 \times (10^4)^{1/2}$ seconds $\longleftrightarrow \approx 3$ min

12 places $\quad 2 \times (10^8)^{1/2}$ seconds $\longleftrightarrow \approx 6$ hrs

20 places $\quad 2 \times (10^{16})^{1/2}$ seconds $\longleftrightarrow \approx 6$ years

$\boxed{7.7}$

#1 $\displaystyle\int_1^\infty \frac{x^2}{e^{-x}+x}\,dx$ diverges.

$$f(x) = \frac{x^2}{e^{-x}+x} = \frac{x^2}{\frac{1}{e^x}+x} =$$

As $x \to \infty$, $\displaystyle f(x) \to \frac{x^2}{0+x} = x \to \infty$

That is, $f(x)$ increases + doesn't approach 0. Therefore, the area under the curve gets larger + larger.

#2 $$\int_0^\infty x^2 e^{-x}\,dx = \lim_{a\to\infty} \int_0^a x^2 e^{-x}\,dx$$

Using integration by parts twice yields,

$$\lim_{a\to\infty} \int_0^a x^2 e^{-x}\,dx = \lim_{a\to\infty} -e^{-x}(x^2+2x+2)\Big|_0^a$$

$$= \lim_{a\to\infty} \underbrace{\frac{-(a^2+2a+2)}{e^a}}_{} + 2 = 2$$

$\longrightarrow 0$ since exponential growth is so much faster than a quadratic.

#3 $\int_0^2 \frac{dx}{(x-1)^2} = \int_0^1 \frac{dx}{(x-1)^2} + \int_1^2 \frac{dy}{(x-1)^2}$

$= \lim_{a\to 1^-} \int_0^a \frac{dx}{(x-1)^2} + \lim_{b\to 1^+} \int_b^2 \frac{dx}{(x-1)^2}$

$= \lim_{a\to 1^-} \left[\frac{-1}{x-1} \Big|_0^a \right] + \lim_{b\to 1^+} \left[\frac{-1}{x-1} \Big|_b^2 \right]$

$= \lim_{a\to 1^-} \left(\frac{-1}{a-1} - 1 \right) + \lim_{b\to 1^+} \left(-1 + \frac{1}{b-1} \right)$

$= \infty - 1 - 1 + \infty = \infty$ diverges

$\int_{-1}^{0} \frac{1}{\sqrt{1+x}} dx = \lim_{a\to -1^+} \int_a^0 \frac{1}{\sqrt{1+x}} dx = \lim_{a\to -1^+} 2\sqrt{1+x} \Big|_a^0$

$= \lim_{a\to -1^+} 2 - 2\sqrt{1+a} = 2$ converges

#4

Total # of sick people $= \int_0^\infty 1000\, te^{-0.5t} dt$

$= \lim_{a\to\infty} \int_0^a 2000te^{-0.5t} dt = \lim_{a\to\infty} 2000\left[-2te^{-.5t} - 4e^{-.5t} \Big|_0^a \right]$

$= 2000 \lim_{a\to\infty} \left(\frac{-2a}{e^{\frac{a}{2}}} - \frac{4}{e^{\frac{a}{2}}} + 4 \right) = 8000$

$\downarrow$ 0 $\quad \downarrow$ 0

since linear growth is significantly slower than exponential growth.

The total # of sick people doubles.

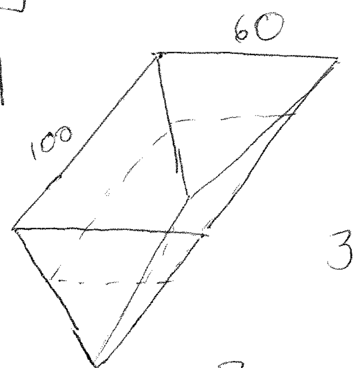
$$\text{Total} = \int_0^\infty 1000te^{-t}\,dt = \lim_{a\to\infty}\int_0^a 1000te^{-t}\,dt$$

$$= 1000\lim_{a\to\infty}\frac{-x-1}{e^x}\Big|_0^a = 1000\lim_{a\to\infty}\left(\frac{-a-1}{e^a}+1\right) = 1000$$

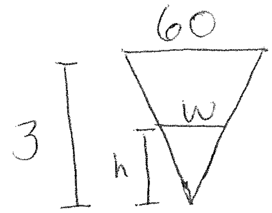
The total # of sick people is $\frac{1}{4}$ as large.

$\boxed{8.1}$

#1

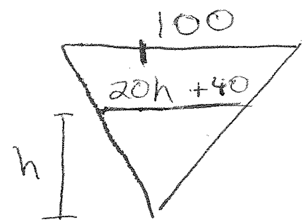


maximum depth $= \frac{30}{10} = 3\text{km}$



$$\frac{w}{60} = \frac{h}{3}$$

$$w = 20h$$



$$\int_0^3 (20h)(20h+40)\,dh = 7200\,\text{km}^3$$

#2

$$4\int_0^1 x^3\sqrt{1-x^2}\,dx = 4\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta\,d\theta =$$

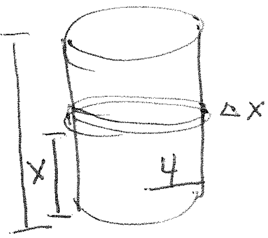
$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$$= 4\int_0^{\frac{\pi}{2}}(1-\cos^2\theta)\sin\theta\cos^2\theta\,d\theta = 4\int_0^{\frac{\pi}{2}} \sin\theta\cos^2\theta - \sin\theta\cos^4\theta\,d\theta$$

$$= 4\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\Big|_0^{\frac{\pi}{2}}\right] = \frac{8}{15}$$

#3

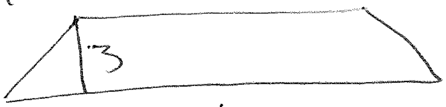


$$\lim_{n\to\infty}\sum_{i=1}^{n} 16\pi\,\Delta x \quad \text{or} \quad \lim_{\Delta x\to 0}\sum_{i=1}^{n} 16\pi\,\Delta x$$

$$16\pi\int_0^h dx = 16\pi x\Big|_0^h = 16\pi h = 112\pi$$

$$h = 7$$

#4



$$\int_0^H \tfrac{1}{2}(16+12)3\,dh = \int_0^H 42\,dh$$

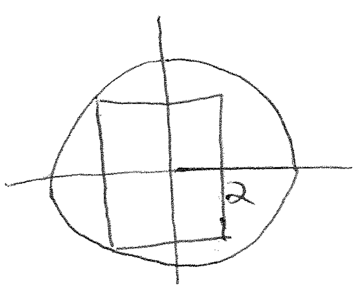
$$= 42h\Big|_0^H = 42H$$

$$42H = 1029$$

$$H = \frac{49}{2} = 24.5\text{cm}$$

$\boxed{8.2}$

#1



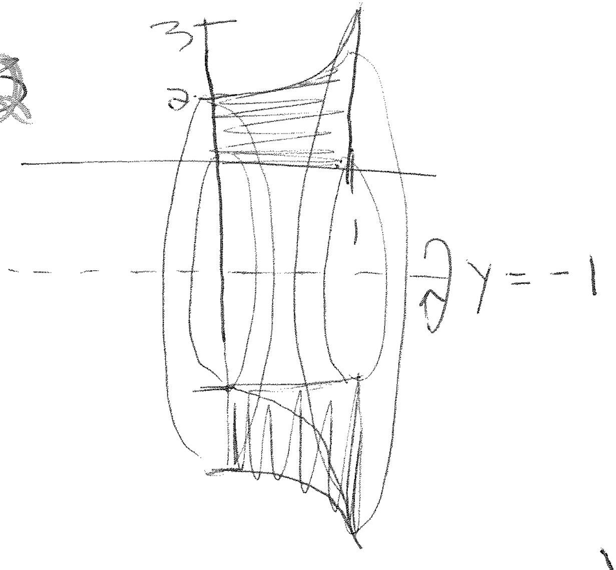
$$\int_{-2}^{2}\left(2\sqrt{4-x^2}\right)^2 dx = \int_{-2}^{2} 4(4-x^2)\,dx$$

$$= \int_{-2}^{2} 16-4x^2\,dx = 16x - \frac{4x^3}{3}\Big|_{-2}^{2} = \frac{128}{3}$$

$y = \pm\sqrt{4-x^2}$

$=$

#2



$$\pi \int_0^1 R^2 - r^2 \, dx$$

$$= \pi \int_0^1 (x^3+2+1)^2 - (1)^2 \, dx$$

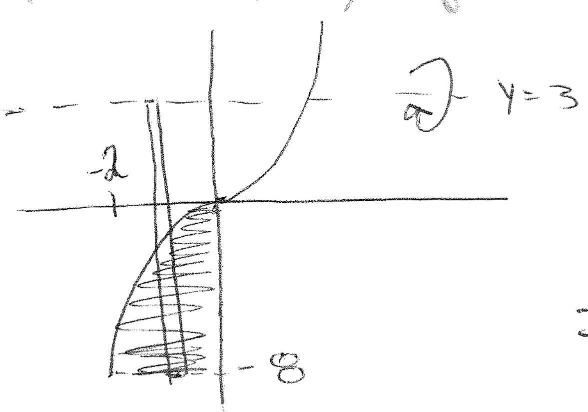
$$= \pi \int_0^1 (x^3+3)^2 - 1 \, dx$$

$$= \pi \int_0^1 x^6 + 6x^3 + 8 \, dx = \pi \left[\frac{x^7}{7} + \frac{6x^4}{4} + 8x \Big|_0^1 \right]$$

$$= \frac{135\pi}{14}$$

#3 (see next page)

#4

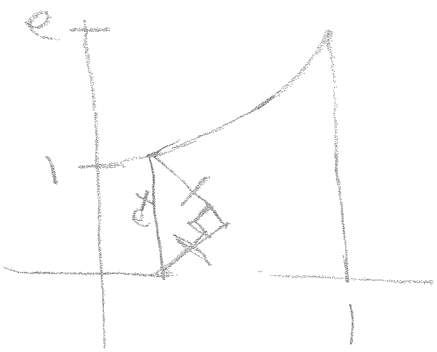


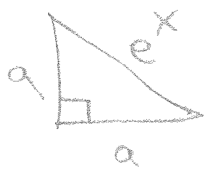
$$\pi \int_{-2}^{0} (11)^2 - (3-x^3)^2 \, dx$$

$$= \pi \int_{-2}^{0} 121 - (9 - 6x^3 + x^6) \, dx$$

$$= \pi \int_{-2}^{0} -x^6 + 6x^3 + 112 \, dx = \pi \left[-\frac{x^7}{7} + \frac{6x^4}{4} + 112x \Big|_{-2}^{0} \right] = \frac{1272\pi}{7}$$

3 (from 8.2)





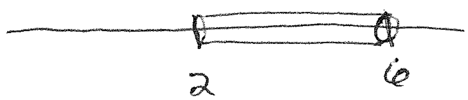
$$a^2 + a^2 = e^{2x}$$
$$2a^2 = e^{2x}$$
$$a^2 = \frac{e^{2x}}{2}$$
$$a = \frac{e^x}{\sqrt{2}}$$

$$A = \frac{1}{2}(a^2) = \frac{1}{2} \cdot \frac{e^{2x}}{2} = \frac{e^{2x}}{4}$$

$$\int_0^1 \frac{e^{2x}}{4}\,dx = \frac{e^{2x}}{8}\Big|_0^1 = \frac{e^2}{8} - \frac{1}{8} = \frac{e^2 - 1}{8}$$

8.4

#1



$$m = \int_2^6 \delta(x)\,dx = \int_2^6 \sqrt{x+1}\,dx$$

$$= \frac{2}{3}(x+1)^{3/2}\Big|_2^6 = \frac{2}{3}\left[7^{3/2} - 3^{3/2}\right]$$

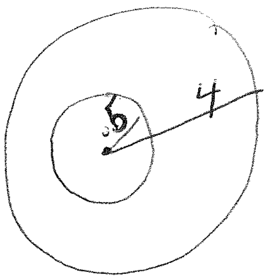
$$\approx 8.88 \text{ grams}$$

$$\bar{x} = \frac{\int_2^6 x\sqrt{x+1}\,dx}{\int_2^6 \sqrt{x+1}\,dx} = \frac{\frac{2}{3}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2}\Big|_2^6}{8.88}$$

integration by parts

$$= 4.14 \text{ cm}$$

#2



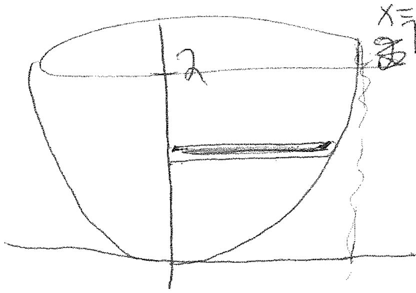
$$4000\int_{.5}^{4}(5-r)(2\pi)r\,dr = 8000\pi\int_{.5}^{4} 5r - r^2\,dr$$

$$= 8000\pi\left[\frac{5r^2}{2} - \frac{r^3}{3}\Big|_{.5}^{4}\right] = 8000\pi\left[40 - \frac{64}{3} - \frac{5}{8} + \frac{1}{24}\right]$$

$$= \frac{434000\pi}{3}$$

$$\approx 454{,}484$$

#3



$$\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left(\sqrt{\frac{y_i}{2}}\right)^2 8(2-y_i)\,\Delta y$$

$$8\pi\int_0^2 \frac{y}{2}(2-y)\,dy = 8\pi\int_0^2 y-\frac{y^2}{2}\,dy = 8\pi\left[\frac{y^2}{2}-\frac{y^3}{6}\Big|_0^2\right] = \frac{16\pi}{3}$$

#4

$$0 = \frac{\sum m_i x_i}{\sum m_i} = \frac{4(-6+1+3+x)}{16}$$

$$4(-6+1+3+x) = 0$$

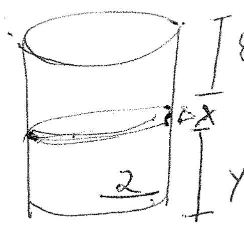
$$-6+1+3+x = 0$$

$$-2+x = 0$$

$$x = 2$$

$\boxed{8.5}$

#1



$\delta = 1000\ kg/m^3$

$$W = \int_0^4 (1000\cdot 9.8)(4\pi)(8-y)\,dy$$

$$= 39200\pi\int_0^4 8-y\,dy$$

$$= 940800\pi\ J$$

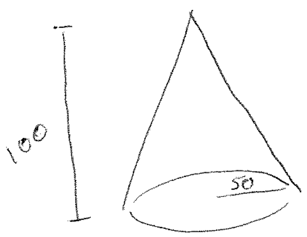
#2

Bottom: $F = \underbrace{62.4\,lb/ft^3(2ft)}_{\text{presure}}\underbrace{(12\,ft^2)}_{\text{area}} = 1497.6$ lbs.

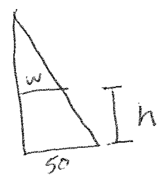
Front & Back: $F = (62.4)(4)\int_0^2 (2-x)\,dx = 499.2$ lbs

Both Sides: $F = (62.4)(3)\int_0^2 (2-x)\,dx = 374.4$ lbs.

#3



$\delta = 2\,lb/ft^3$



$\frac{w}{50} = \frac{100-h}{100}$

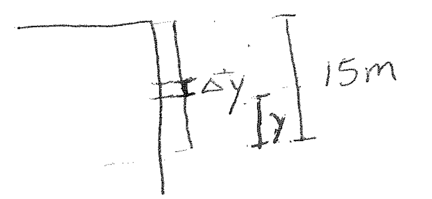
$w = 50 - \frac{h}{2}$

$$\pi\int_0^{100}\left(50-\frac{h}{2}\right)^2\cdot 2\cdot h\,dh = 2\pi\int_0^{100} 2500h - 50h^2 + \frac{h^3}{4}\,dh$$

$$= 2\pi\left[\frac{2500h^2}{2} - \frac{50h^3}{3} + \frac{h^4}{16}\Bigg|_0^{100}\right] = \frac{12500000\pi}{3} \approx 1.3\times 10^7 \text{ ft-lbs}$$

#4

$F_1 \overset{\text{per meter}}{=} mg = (4\,kg)(9.8\,m/s^2) = 39.2\,N$



$$\int_0^{15} 39.2(15-y)\,dy = 39.2\left(15y - \frac{y^2}{2}\right)\Bigg|_0^{15}$$

$$= 4410\,J$$

$$\int_0^{30} 39.2(30-y)\,dy = 39.2\left(30y - \frac{y^2}{2}\right)\Bigg|_0^{30}$$

$$= 17640\,J$$

The work is 4 times as much.